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		– Nielsen & Chuang		

Ex 8.29 Amplitude damping of single qubit density matrix

For the general single qubit state

$$\rho = \begin{bmatrix} a & b \\ b^* & c \end{bmatrix}$$

show that amplitude damping leads to

$$\mathcal{E}_{AD}(\rho) = \begin{bmatrix} 1-(1-\gamma)(1-a) & b\sqrt{1-\gamma} \\ b^*\sqrt{1-\gamma} & c(1-\gamma) \end{bmatrix}$$

Ans: $E_0 = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-\gamma} \end{bmatrix}$, $E_1 = \begin{bmatrix} 0 & \sqrt{\gamma} \\ 0 & 0 \end{bmatrix}$

$$\mathcal{E}_{AD}(\rho) = \sum_k E_k \rho E_k^\dagger$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-\gamma} \end{bmatrix}\begin{bmatrix} a & b \\ b^* & c \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-\gamma} \end{bmatrix} + \begin{bmatrix} 0 & \sqrt{\gamma} \\ 0 & 0 \end{bmatrix}\begin{bmatrix} a & b \\ b^* & c \end{bmatrix}\begin{bmatrix} 0 & 0 \\ \sqrt{\gamma} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} a & b \\ b^*\sqrt{1-\gamma} & c\sqrt{1-\gamma} \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-\gamma} \end{bmatrix} + \begin{bmatrix} b^*\sqrt{\gamma} & c\sqrt{\gamma} \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ \sqrt{\gamma} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} a & b\sqrt{1-\gamma} \\ b^*\sqrt{1-\gamma} & c(1-\gamma) \end{bmatrix} + \begin{bmatrix} c\gamma & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} a+c\gamma & b\sqrt{1-\gamma} \\ b^*\sqrt{1-\gamma} & c(1-\gamma) \end{bmatrix}$$

$$tr(\rho) = 1 \implies a+c = 1 \implies c = (1-a)$$

$\therefore$

$$\mathcal{E}_{AD}(\rho) = \begin{bmatrix} a+(1-a)\gamma & b\sqrt{1-\gamma} \\ b^*\sqrt{1-\gamma} & c(1-\gamma) \end{bmatrix}$$

$$= \begin{bmatrix} \cancel{a}-1+1+(1-a)\gamma & b\sqrt{1-\gamma} \\ b^*\sqrt{1-\gamma} & c(1-\gamma) \end{bmatrix}$$

$$= \begin{bmatrix} 1-(1-a)(1-\gamma) & b\sqrt{1-\gamma} \\ b^*\sqrt{1-\gamma} & c(1-\gamma) \end{bmatrix}$$

$$= \begin{bmatrix} 1-(1-\gamma)(1-a) & b\sqrt{1-\gamma} \\ b^*\sqrt{1-\gamma} & c(1-\gamma) \end{bmatrix}$$

Ex. 8.23

Suppos[e]
by us[ing]
Section 7.4.1

$|$

Show
gives
by th[e]

$E_0^{d\tau} =$

$E_1^{d\tau} =$

i.e., ei[ther]
qubit,
into
to $|\psi\rangle$

Q. Stack 2/2/22

Why
error
dampin[g]

Ex: 8.23 Amplitude damping of dual-rail qubits

Suppose that a single qubit state is represented by using 2 qubits, as

Section 7.4.1

$$|\psi\rangle = a|01\rangle + b|10\rangle$$

Show that $\mathcal{E}_{AD} \otimes \mathcal{E}_{AD}$ applied to this state gives a process which can be represented by the operation elements

$$E_0^{dr} = \sqrt{1-\gamma}\ I$$

$$E_1^{dr} = \sqrt{\gamma}\left[|00\rangle\langle 01| + |00\rangle\langle 10|\right]$$

i.e., either nothing (E_0^{dr}) happens to the qubit, or the qubit is transformed (E_1^{dr}) into the state $|00\rangle$, which is orthogonal to $|\psi\rangle$.

Q. Stack 2/12/22

Why this dual-rail encoding is called an error detecting code for the amplitude damping channel?

Ans:

$\mathcal{E}_{AD}$ represents amplitude damping such that

$\mathcal{E}_{AD}(\rho) = E_0 \rho E_0^\dagger + E_1 \rho E_1^\dagger$, where

$$E_0 = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-\gamma} \end{bmatrix} = |0\rangle\langle 0| + \sqrt{1-\gamma}\,|1\rangle\langle 1| \quad \text{and}$$

$$E_1 = \begin{bmatrix} 0 & \sqrt{\gamma} \\ 0 & 0 \end{bmatrix} = \sqrt{\gamma}\,|0\rangle\langle 1| \quad , \text{ and}$$

$$\mathcal{E}\left(\begin{bmatrix} a & b \\ b^* & c \end{bmatrix} \right) = \begin{bmatrix} 1-(1-\gamma)(1-a) & b\sqrt{1-\gamma} \\ b^*\sqrt{1-\gamma} & c(1-\gamma) \end{bmatrix}$$

$$(\mathcal{E}_{AD} \otimes \mathcal{E}_{AD})(\rho \otimes \sigma) = \mathcal{E}_{AD}(\rho) \otimes \mathcal{E}_{AD}(\sigma)$$

$$= \left(E_0 \rho E_0^{\dagger} + E_1 \rho E_1^{\dagger}\right) \otimes \left(E_0 \sigma E_0^{\dagger} + E_1 \sigma E_1^{\dagger}\right)$$

$$= E_0 \rho E_0^{\dagger} \otimes E_0 \sigma E_0^{\dagger} + E_0 \rho E_0^{\dagger} \otimes E_1 \sigma E_1^{\dagger} + E_1 \rho E_1^{\dagger} \otimes E_0 \sigma E_0^{\dagger} + E_1 \rho E_1^{\dagger} \otimes E_1 \sigma E_1^{\dagger}$$

$$= (E_0 \otimes E_0)(\rho \otimes \sigma)(E_0 \otimes E_0)^{\dagger} + (E_0 \otimes E_1)(\rho \otimes \sigma)(E_0 \otimes E_1)^{\dagger} + (E_1 \otimes E_0)(\rho \otimes \sigma)(E_1 \otimes E_0)^{\dagger} + (E_1 \otimes E_1)(\rho \otimes \sigma)(E_1 \otimes E_1)^{\dagger}$$

Let $|0\rangle_D = |01\rangle$ and $|1\rangle_D = |10\rangle$,

$$(E_0 \otimes E_0)|1\rangle_D = \sqrt{1-\gamma}\,|1\rangle_D$$

$(E_0 \otimes E_1)|01\rangle = \sqrt{\gamma}\,|00\rangle_D$ and $(E_1 \otimes E_0)|10\rangle = \sqrt{\gamma}\,|00\rangle$

$(E_0 \otimes E_1)|10\rangle = 0$ and $(E_1 \otimes E_0)|01\rangle = 0$

$$(E_1 \otimes E_1)|1\rangle_D = 0$$

$$|\psi\rangle = a|01\rangle + b|10\rangle$$

$$|\psi\rangle\langle\psi| = (a|01\rangle + b|10\rangle)(a^*\langle 01| + b^*\langle 10|)$$

$$= |a|^2|01\rangle\langle 01| + ab^*|01\rangle\langle 10| + ba^*|10\rangle\langle 01| + |b|^2|10\rangle\langle 10|$$

$$(\mathcal{E}_{AD}\otimes\mathcal{E}_{AD})(|\psi\rangle\langle\psi|) = (1-\gamma)(|\psi\rangle\langle\psi|) + (|a|^2+|b|^2)(|00\rangle\langle 00|)$$

$$= (1-\gamma)(|\psi\rangle\langle\psi|) + \gamma|00\rangle\langle 00|$$

$$E_0^{dr}(|\psi\rangle\langle\psi|)E_0^{dr\dagger} + E_1^{dr}(|\psi\rangle\langle\psi|)E_1^{dr\dagger} = (1-\gamma)(|\psi\rangle\langle\psi|) + \gamma|00\rangle\langle 00|$$

The
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(automatically)
The dual-rail encoding provides error detection capabilities :

If a state with no photons is detected, we know that a loss event has taken place.

If operating in this so-called error-detection mode, the receiver must signalize such event, and request the signal to be transmitted anew.

$$\mathcal{E}_{AD}\left(\begin{bmatrix} a & b \\ b^* & c \end{bmatrix}\right) = \begin{bmatrix} a + c\gamma & b\sqrt{1-\gamma} \\ b^*\sqrt{1-\gamma} & c(1-\gamma) \end{bmatrix}$$

$$\rho = \frac{1}{2}\left(I + \vec{r}\cdot\vec{\sigma}\right) = \frac{1}{2}\begin{bmatrix} 1+z & x-iy \\ x+iy & 1-z \end{bmatrix}$$

$$\mathcal{E}_{AD}(\rho) = \frac{1}{2}\begin{bmatrix} 1+z+\gamma(1-z) & \sqrt{1-\gamma}(x-iy) \\ \sqrt{1-\gamma}(x+iy) & (1-\gamma)(1-z) \end{bmatrix}$$

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},\ Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix},\ Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\mathcal{E}_{AD}(\rho) = \frac{1}{2}\begin{bmatrix} 1+\gamma+z(1-\gamma) & \sqrt{1-\gamma}\,x - i\sqrt{1-\gamma}\,y \\ \sqrt{1-\gamma}\,x + i\sqrt{1-\gamma}\,y & 1-\gamma-(1-\gamma)z \end{bmatrix}$$

$$= \frac{1}{2}\left[I + x\sqrt{1-\gamma}\,X + y\sqrt{1-\gamma}\,Y + \left(\gamma + z(1-\gamma)\right)Z\right]$$

$\vec{r} = (x, y, z) \xrightarrow{\mathcal{E}} \vec{r}' = \left(x\sqrt{1-\gamma},\ y\sqrt{1-\gamma},\ \gamma + z(1-\gamma)\right)$

⇒ The effect of the amplitude damping channel on the Bloch sphere, is that the entire sphere shrinks towards the north pole; the $|0\rangle$ state.